W03

Proceedings of the 5th International Conference on Inverse Problems in Engineering: Theory and Practice, Cambridge, UK, 11-15th July 2005

EFFECT OF DETERMINISTIC THERMOCOUPLE ERRORS (BIAS) ON THE SOLUTION OF THE INVERSE HEAT CONDUCTION PROBLEM

K. A. WOODBURY and A. GUPTA
Department of Mechanical Engineering, The University of Alabama, Tuscaloosa, AL, 35487, USA
e-mail: woodbury@me.ua.edu

Abstract - This paper demonstrates the deterministic errors in thermocouple measurements, or bias, through numerical simulation and illustrates the impact of these erroneous measurements on inverse heat conduction problem solutions. The case of molten metal solidifying through cooling in a sand mold is considered. Artificial data for the sand surface temperature and at two different locations below the sand surface are obtained through numerical simulation. Temperature data obtained from these simulations are used as input to the inverse heat conduction problem to determine the surface heat flux as a function of time. Results for four different thermocouple diameters are presented.

1. INTRODUCTION

Thermocouples are the most widely used temperature-measuring device for monitoring temperature during casting of metal parts. Three possible thermocouple installations are of interest: (i) imbedded within or attached to the casting itself, (ii) mounted flush to the casting to measure the mold wall temperature, and (iii) imbedded below the mold surface to measure the temperature of the mold. A review of the literature reveals that similar problems of temperature measurement using thermocouples have been known for decades, but such problems are still being studied.

Tszeng and Saraf [1] studied a "fin effect" associated with the surface mounted thermocouples. Errors are introduced in measurements as the heat conduction into the thermocouple changes the local temperature at the thermocouple junction depending upon the wire diameter [1]. A computational model using a thermocouple of 0.4 mm diameter and 10 mm in length was used to calculate the temperature at the junction of thermocouple wire. A Jominy end quench test was performed with type K thermocouples of 0.25 mm diameter attached to the surface to measure the temperatures. Using these measurements and an inverse technique, the surface heat transfer coefficient and overall transient temperature field were determined. The thermocouple junction temperatures calculated from the simulation agreed well with the measured temperature.

Park *et al.* [2] did a study of surface mounted thermocouples and considered the error introduced by heat conduction along the leads of a thermocouple during rapid transient cooling. Through numerical calculations, they demonstrated that, during boiling at the surface, conduction through the thermocouple wires creates a significant depression of the surface temperature at the attachment points. As a result of this temperature depression, determination of the local (undisturbed) surface temperature is difficult. The authors present a parameter estimation technique to obtain the surface heat flux and undisturbed surface temperature. One important conclusion of this study was that their 2-D and 3-D model studies show that surface temperature measured using such an intrinsic thermocouple may be in error by 40 C due to lead heat loss [2].

Attia *et al.* [3–6] performed a long-term, comprehensive study of thermocouple errors due to conduction. They developed a model for estimating the systematic or deterministic temperature measurement error due to thermal disturbance in the region around the thermocouple junction. A three-dimensional finite element model for a thermocouple installed in a blind hole of a block was studied for types E, J and T (AWG 24, 30) thermocouples. Different cases were studied by varying mean thermal conductivity of thermocouple, thermal conductivity of filler material, position of thermocouple in the hole and temperature gradient across the block. It was found that the pattern of the disturbed temperature field in the region surrounding the thermocouple was dependent on the ratio between the thermal conductivities of the filler material and the block. Further it was concluded that reduction in the temperature gradient in the undisturbed field significantly increases the heat flow "leak" into thermocouple wires resulting in a considerable systematic temperature errors. Results for eccentric positioning of the thermocouple showed that when the thermocouple is positioned towards the hot side of the hole temperatures measured by the hot junction were biased and overestimated i.e. significant temperature measurement errors resulted [6]. Similarly when the thermocouple was shifted towards the cold side of the hole the temperatures measured were biased and underestimated [6]. Experimental verification using a well-controlled experiment validated these results.

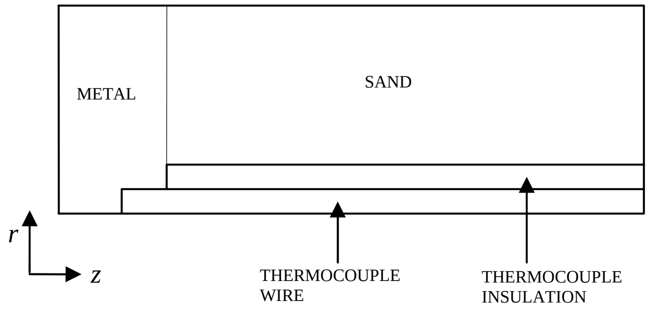

Figure 1. Outline of axisymmetic model.

2. MODEL DESCRIPTION

A beaded thermocouple consists of two wires of dissimilar metals that are joined to form a junction, typically by welding the ends together. As mentioned in the Introduction, three types of thermocouple installations are of interest for a casting process: (i) metal surface, (ii) mold surface, and (iii) mold subsurface. A computational model was developed to simulate the errors associated with thermocouple installations. The results from these simulations are used later as input into an inverse heat conduction analysis.

In the present study, a simple two dimensional finite element computational model was developed to simulate thermocouple installations. The commercial finite element package FIDAP™ was used (version 7.0). A simple case of molten metal solidifying in a sand mold was considered for this purpose. The model is an axisymmetric model with the thermocouple running across the center.

An outline of the model is shown in Figure 1 showing the molten metal, sand mold, thermocouple, and thermocouple insulation for the case of an intrinsic metal surface measurement. The length of metal and sand zone is 12 mm and 120 mm, respectively, and the outer radius of the domain surrounding the insulation and thermocouple is 10 mm. Thus, Figure 1 depicts on-half of a cylindrical core which has a thermocouple wire surrounded by thermocouple insulation and sand, with a disk of solidifying metal to the left.

For simplification, the thermocouple is considered as a single wire having an effective diameter that gives the same cross sectional area as the actual two wires. This is similar to the approach used by Attia and Kops [4], and Sparrow [11] and others. Effective properties of the two wires were applied to this simplified thermocouple. A parallel combination was used for the thermal conductivity and a mass averaged value for the product ρC_p.

The insulation thickness was determined by measurement for an AWG 24 thermocouple with glass braid insulation and this thickness was used for all of the cases studied. Four different thermocouple gages (AWG 24, 30, 36, 44) were considered in the analysis. Also, two different thermocouple insulations (Glass braid and Al_2O_3) were studied for each thermocouple.

Throughout the paper, thermocouples are referred to according to the AWG number. Table 1 shows the correspondence of diameter to AWG number for the wires used in this paper.

Table 1. Nominal diameters corresponding to American Wire Gauge (AWG) numbers.

AWG	Diameter, mm
24	0.51
30	0.25
36	0.13
44	0.05

Each case was simulated for a period of 500 seconds. The molten metal considered was aluminum and the initial pour temperature was taken as 670 C. Although the metal is molten for part of the simulation time, no motion of the molten metal was considered and so the simulation is for conduction only. The initial temperature of the sand and thermocouple was taken as 20 C. A contact conductance (heat transfer coefficient) of 300E-06 W/mm-K was considered for the gap between the molten metal and the sand mold.

For each combination of wire and insulation studied the three different types of temperature measurements were considered: (i) metal surface, (ii) mold surface, and (iii) mold subsurface. For the case when metal temperature was measured the thermocouple was inserted 10mm inside the metal. For the case of mold surface temperature measurement the thermocouple was flush with the surface of the mold. For the case of subsurface temperature measurements, two different locations of the thermocouple from the surface of mold, 5 mm and 10 mm, were considered.

Properties used for the analysis for metal, sand, thermocouple wire, insulation are given in Table 2. Note that for most of the materials, the density was not specified separately but was combined with the specific heat. However, for the molten and solidifying metal, the specific heat was computed from an enthalpy model with enthalpy data specified via a curve fit, and so the density is specified separately for this case.

All boundaries in the model are considered adiabatic. The extent of the domain in the *z* direction is large and during the time of simulation the heat given up by the solidifying metal does not effect a temperature rise at the right end of the domain in Figure 1.

Table 2: Thermal properties used for the computation.

	Density (Kg/mm^3)	Conductivity (W/mm-K)	ρC_p (J/mm^3-K)
Metal	2770E-09	180E-03	curve fit
Sand	1.0	0.1E-02	0.14E-02
Wire	1.0	0.228E-01	0.385E-02
Glass braid	1.0	0.36E-04	0.835E-04
Al_2O_3	1.0	0.15E-01	0.303E-02

3. COMPUTATIONAL DETAILS

Computations were performed for the three types of installations (metal surface, mold surface, and mold subsurface), and two types of thermocouple insulation were considered for each case (glass braid and Al_2O_3). From the simulations, the results for the temperature at the thermocouple tip can be compared to the corresponding undisturbed temperature (at the same *z* location but at the outer radius). This difference will be the deterministic error, or bias, due to the presence of the thermocouple.

An important task was to make the model grid independent. Grid independence means that the converged solution obtained from a calculation is independent of the grid density; i.e. increasing the number of cells would not change the solution. In practice, grid independence is achieved when further refinement of mesh yields negligible or insignificant changes in the solution of the model. For the present case, the model was initially solved for a coarse mesh. Once the model was converged the mesh was refined by increasing the number of grid points and then re-solved. A comparison between the two solutions was made by looking at the temperature history in the region of the thermocouple tip, since that region is of prime importance in this investigation. The RMS error between the two solutions was calculated as follows:

$$RMS_{ERROR} = \sqrt{\frac{\sum_{i=1}^{N}(T_i - T_i^{/})^2}{N}} \tag{1}$$

where N is the number of points in the time history, T_i is the temperature at the thermocouple tip, and T_i' is the temperature on the refined grid. The mesh refinement was stopped when this RMS error became about 0.0121 C, resulting in the smallest element having a size of 0.25×0.04 mm for AWG 24 and 0.25×0.0042 mm for AWG 44.

The mesh near the interface of the molten metal and mold was made denser since this is where the temperature gradients are largest. However, increasing the grid points also increases the computational time, hence to reduce the computational time a graded mesh was used, whereby the mesh at the end of sand mold (where temperature gradients are low) was made coarse. The typical mesh was 36 nodes by 90 nodes.

The phase change in FIDAP was obtained using the enthalpy model, as mentioned earlier. A common means for inclusion of the latent heat in the material properties is enthalpy/specific heat methods. Enthalpy values were supplied as a function of temperature, and linear interpolation was performed to obtain the enthalpy at the intermediate points on the curve. The curve is extended at least 10-20% above and below the expected temperature ranges to account for temperatures that may arise in the nonlinear iterations before a converged solution is attained.

Typical solution times on a Pentium4, 2GHz, 1Gb RAM machine are about 60 minutes.

4. COMPUTED THERMOCOUPLE ERRORS

To understand the nature of these deterministic errors, consider the results from one of the computations shown in Figure 2. The figure shows a magnified view of the temperature field in the vicinity of the thermocouple tip. The thermocouple wire and surrounding insulation are at the bottom of the figure depicted by the mesh. The upper portion of the mesh represents the insulation around the wire; note the change in the slope at the wire/insulation interface due to the difference in thermal conductivities of the two materials.

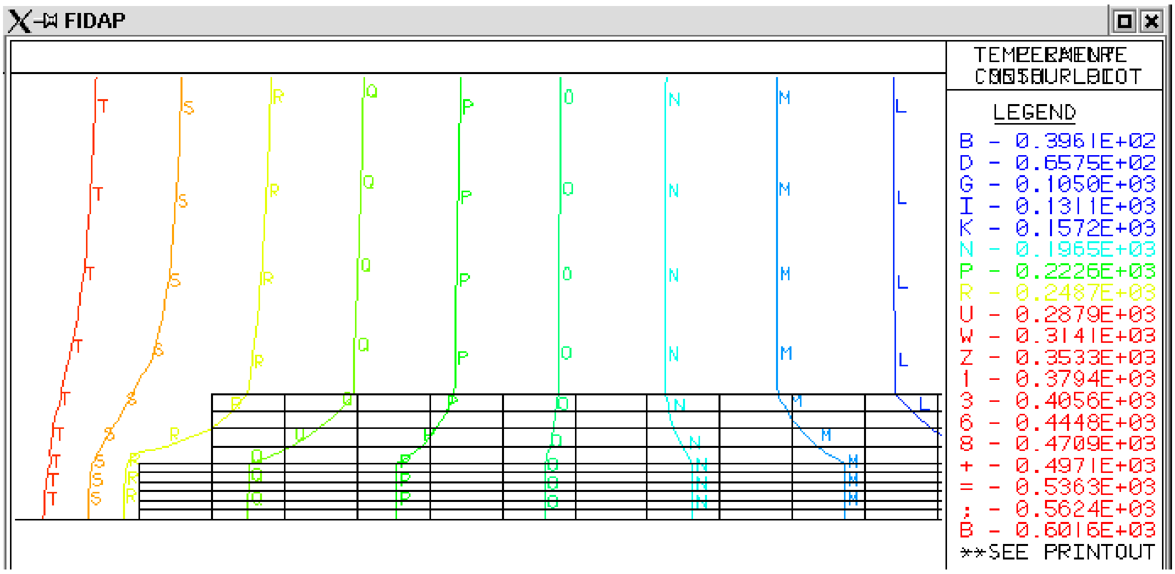

Figure 2. Closeup view of the isotherms distorted by the presence of a thermocouple.

The deterministic thermocouple error can easily be visualized in Figure 2. The temperature near the tip of the thermocouple is at about the level of the isotherm 'R' in the figure, but the "true" temperature at the tip location should be at about the level of the isotherm 'S' (directly above the tip and away from the distortion). This "true" temperature may also be referred to as the "estimated undisturbed" temperature.

To quantify the thermocouple error, the average temperature of the tip was found by volume-averaging the values from the elements comprising the tip. This value is used as the "measured" value. The value of temperature at the same axial location, but farthest away from the tip (at the maximum radial location) is taken as the "true" value. The difference between the "true" and "measured" values is the deterministic error, or bias, in the measurement.

4.1 Case I: Metal Surface Temperature

In practice, the surface temperature of the solidifying metal is obtained by inserting an exposed thermocouple wire into the mold cavity during the construction of the mold. This is accomplished by imbedding the thermocouple wire in the mold and allowing the bare wires to protrude from the mold wall into the mold cavity. When molten metal is introduced into the cavity at the initiation of casting, the molten metal freezes onto the bare wire, forming an electrical junction at the surface of the metal. This intrinsic junction affords a "two-point" thermocouple measurement which is the average temperature of the thermocouple wire at the surface of the metal.

To simulate this measurement, the thermocouple was inserted 10 mm inside the molten metal, and the metal was allowed to solidify. In this case, the error is defined as the difference in the average temperature of the wire at the metal surface and that of the temperature of a node on the metal surface located above the wire on the top edge of the model, see Figure 1.

The difference between these temperature measurements is the deterministic thermocouple error. These errors were computed for each of the four thermocouple diameters (AWG 24, 30, 36, and 44) and for the two different insulations. Figure 3 shows a graph of these errors for all the four thermocouple diameters with glass braid insulation. Similarly, Figure 4 shows errors for all the thermocouples with Al_2O_3 insulation.

4.2 Case II: Mold Surface Temperature

For the case of the mold surface measurement, the thermocouple was placed flush with the surface of the mold. In this case the volume-averaged tip temperature was compared to the surface temperature at the topmost point of the domain. Figure 5 and Figure 6 show the errors for surface temperature measurement for both glass braid and Al_2O_3 insulation for the four different thermocouples.

4.3 Case III: Mold Subsurface Temperature

For the case of subsurface temperature measurement, two different thermocouple locations were considered: 5 mm and 10 mm below the mold surface. Figure 7 and Figure 8 show the subsurface errors for both glass braid and Al_2O_3 insulations when the thermocouple was placed 5 mm from the surface of the mold.

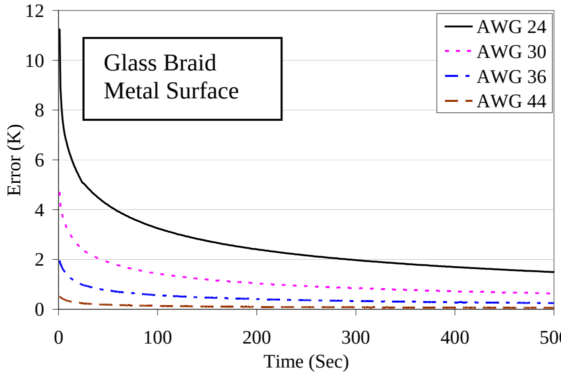

Figure 3. Errors with glass braid insulation for metal surface temperature measurements.

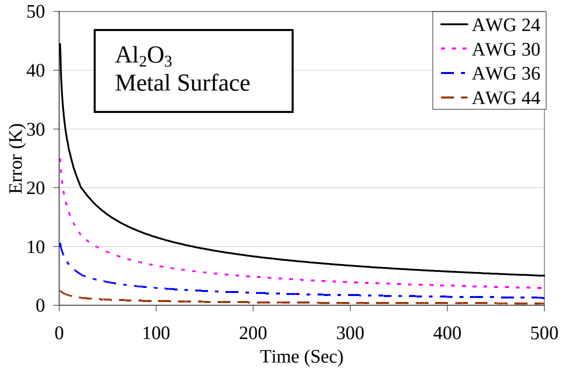

Figure 4. Errors with Al_2O_3 insulation for metal surface temperature measurements.

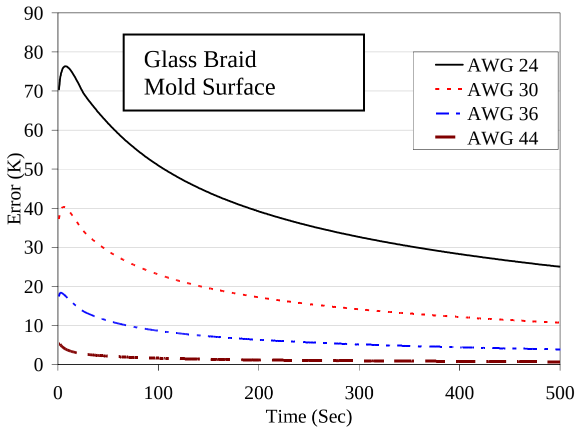

Figure 5. Errors with glass braid insulation for mold surface temperature measurements.

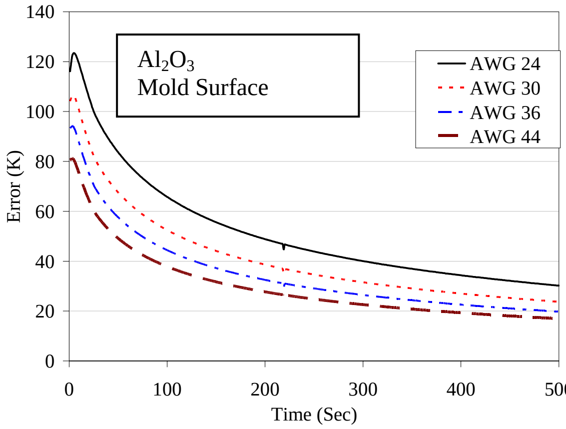

Figure 6. Errors with Al_2O_3 insulation for mold surface temperature measurement.

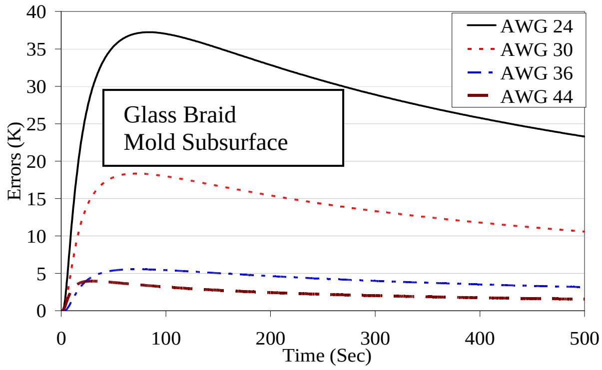

Figure 7. Errors with glass braid insulation for sub-surface (5 mm) temperature measurements.

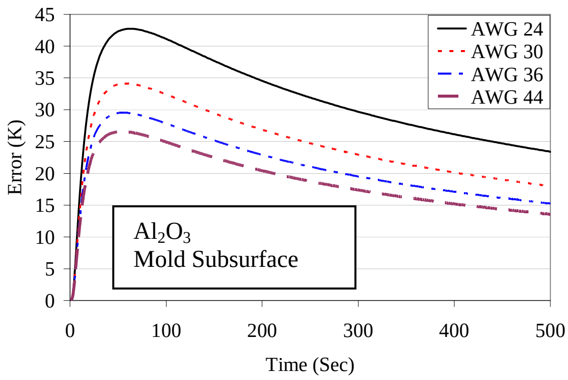

Figure 8. Errors with Al_2O_3 insulation for sub-surface (5 mm) temperature measurements.

5. INVERSE PROBLEM

Solution to an inverse problem can be described as the determination of unknown causes based on observation of their effects [9]. In the present study, the unknown cause to be determined is the heat flux history which is to be determined based on the measured surface or subsurface temperature. For this function estimation problem, the unknown parameter is the heat flux which is a function of time. For a one-dimensional transient conduction problem, the equations governing the flow of heat in the mold surrounding the metal are given by:

$$C(T)\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k(T)\frac{\partial T}{\partial x}\right) \tag{2}$$

with initial condition and boundary conditions as

$$T(x, 0) = g(x) \tag{3}$$

$$-k \left.\frac{\partial T}{\partial x}\right|_{x=L} = 0 \tag{4}$$

$$-k \left.\frac{\partial T}{\partial x}\right|_{x=0} = q(t) \tag{5}$$

Here the function $q(t)$ is to be determined, based on temperature measurements at one or more points in the domain. The function specification method to solve this inverse problem was proposed by Beck [9]. The simplest form is the constant heat flux function form in which it is temporarily assumed that for several future time steps the heat fluxes are constant. Thus heat flux components $\hat{q}_1$, $\hat{q}_2$ ….. $\hat{q}_{M-1}$ are assumed to be known and the task is to estimate $\hat{q}_M$. To add stability to the algorithm the heat flux components q_M, q_{M+1}, …, q_{M+r-1} are assumed to be equal thereby making "*r*" future flux values temporarily equal. Expressions for T_M, T_{M+1} ….. T_{M+r-1} are obtained from a discrete form of Duhamel's integral [9]. Furthermore, the least square procedure for estimation of q_M with temperature measurements Y_M, Y_{M+1}, ….. Y_{M+r-1}, minimizes the following with respect to q_M

$$S = \sum_{i=1}^{r} (Y_{M+i-1} - T_{M+i-1})^2 \tag{6}$$

Differentiating eqn. (6) with respect to q_M and equating to zero and replacing q_M with its estimate $\hat{q}_M$ we get the function specification equation

$$\hat{q}_M = \frac{\sum_{i=1}^{r} (Y_{M+i-1} - Y_{M+i-1})\phi_i}{\sum_{i=1}^{r} \phi_i^2}. \tag{7}$$

Here the ϕ_i are the sensitivity coefficients which quantify the rate of change in the measured temperature with respect to the unknown heat flux. Larger values of "*r*" give more stable solutions by reducing the sensitivity to measurement errors. For the present analysis, the "*r*" value was taken as 5 for all the cases. A MatLAB® code was written for the above algorithm for which inputs (measured temperature, time, location of thermocouple and thermal properties) were supplied to obtain the heat flux at the surface.

6. EFFECT OF ERRORS ON THE INVERSE PROBLEM SOLUTION

The "true" heat flux to the mold during the casting was obtained from the FIDAP simulation using the FLUX command. Heat fluxes obtained from a general finite element program such as FIDAP is sometimes questioned. To test both the accuracy of the heat fluxes produced by FIDAP and the accuracy of the inverse heat conduction solver used in this work, a comparison of the heat fluxes from the two programs was made. For the inverse problem, "true" temperature data (the estimated undisturbed temperature history) at a 5 mm depth was used and a solution obtained using $r = 5$. The results are compared with the FIDAP output in Figure 9, and the two curves are indistinguishable. While it may be argued that both the FIDAP and inverse solution are in error and that happen to agree, it is much more plausible that they are both substantially correct and that they agree.

Figure 10 shows the heat flux history obtained from FIDAP and the flux histories obtained from the solution of inverse problem using data from the mold surface for AWG 24 thermocouple. Both glass braid and Al_2O_3 insulations are shown. Figure 11 shows similar flux histories obtained using subsurface data at 5 mm as input to the inverse heat conduction problem.

7. DISCUSSION

The presence of thermocouples distorts the temperature field and results in an unavoidable measurement error. Figure 2 shows the distortion in the temperature field around a thermocouple. The errors due to the thermocouple's presence are depicted in Figures 3-8. Note that for subsurface temperature measurements, see Figures 7-8, the errors are larger for Al_2O_3 than for glass braid insulation for the 24 AWG, but are generally higher for Al_2O_3 for the smaller diameters. This illustrates that the effect of axial conduction in the wire is much more important when the lateral heat loss is inhibited due to the lower conductivity of Al_2O_3.

The errors in direct metal surface temperature measurements are lowest of the cases considered, but still they are of the order 10 K to 40 K. For measurements in the sand surrounding the metal, where the conductivity of the thermocouple wire is large compared to the conductivity of the medium, the errors are much larger. The errors are largest for the mold surface temperature measurement, of the order 70 K to 120 K. As the measurement point moves away from the active surface, the magnitude of the error diminishes to about 10 K to 40 K, and the time of occurrence of the maximum error increases.

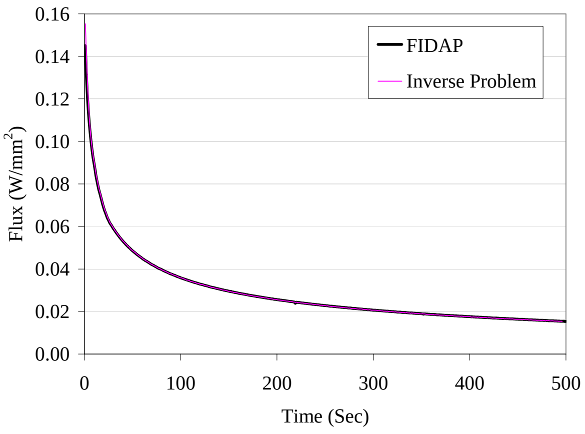

Figure 9. FIDAP flux history and IP solution using "true" data from 5 mm depth.

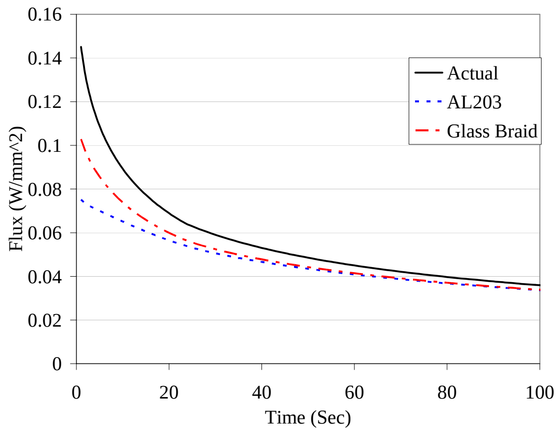

Figure 10. Flux histories from IP solution using data from mold surface (AWG 24).

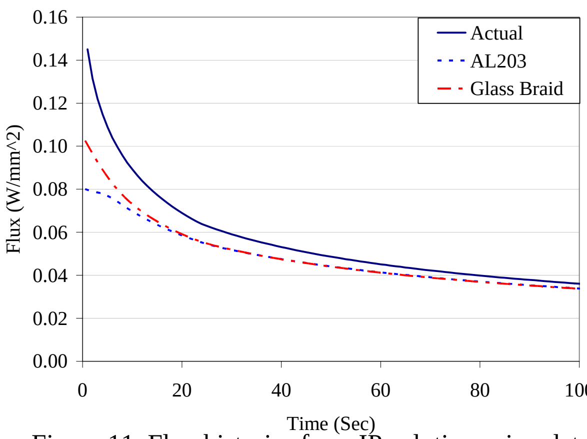

Figure 11. Flux histories from IP solution using data from 5 mm depth (AWG 24).

Of course, the errors diminish as the diameter of the thermocouple decreases. A practical size limit for commercial/industrial applications is about 24 AWG. For 24 AWG thermocouples, measurement errors on the order of 75 C are possible for mold surface temperature measurements, and errors of about 35 C are possible for subsurface measurements.

If these error-laden histories are used in the solution of the inverse problem, errors in the estimated heat flux will result. For the case of the subsurface measurement, initial errors on the order of 40% are possible; see Figure 10, "Al_2O_3" case.

8. CONCLUSIONS

A finite element model has been used to demonstrate the deterministic thermocouple errors, or bias, using four different diameter type K thermocouples with glass braid and Al_2O_3 insulation. The case of solidification of a metal to a surrounding sand mold has been considered. Temperature measurement simulations for metal surface, mold surface and mold subsurface cases were obtained. The results show that deterministic errors in the thermocouples are proportional to the thermocouple wire diameter, i.e. as the diameter of thermocouple wire decreases (or AWG number increases) the deterministic errors for all the cases also decrease. Also, for all the cases of the temperature measurement, the errors were larger for Al_2O_3 insulation than for glass braid insulation due to the much lower thermal conductivity of the Al_2O_3.

Using an inverse technique, the heat flux histories at the surface of the mold were computed and compared with the flux histories obtained from the numerical simulation. Due to the deterministic errors, the flux histories predicted by the inverse technique were underestimated when compared to the flux histories obtained from the numerical simulation.

For the subsurface case when the thermocouple was located 5 mm from the surface it can be seen that the flux histories are more erroneous in the case of Al_2O_3 insulation as compared to glass braid insulation. This is due to the fact that the deterministic errors in the case of Al_2O_3 insulation were more than glass braid insulation due to the difference in the thermal conductivity of these two materials.

REFERENCES

1. T.C. Tszeng and V. Saraf, A study of fin effects in the measurement of temperature using surface-mounted thermocouples. *ASME J. Heat Transfer* (2003) **125**, 926-935.
2. J.E. Park and K.W. Childs, Correction of errors in intrinsic thermocouple signals recoded during quenching, *AIChe Symposium Series*, 1991, pp.309-318.
3. M.H. Attia and L. Kops, Distortion in thermal field around inserted thermocouples in experimental interfacial studies. *Trans. ASME* (1986) **108**, 241-246.
4. M.H. Attia and L. Kops, Distortion in thermal field around inserted thermocouples in experimental interfacial studies-Part II: Effect of the heat flow through the thermocouple. *Trans. ASME* (1988) **110**, 7-14.
5. M.H. Attia and L. Kops, Distortion in thermal field around inserted thermocouples in experimental interfacial studies-Part 3: Experimental and numerical verification. *Trans. ASME* (1993), **115**, 444-449.
6. M.H. Attia, A. Cameron and L. Kops, Distortion in thermal field around inserted thermocouples in experimental interfacial studies, Part 4: End effect. *Trans. ASME* (2002) **124**, 135-145.
7. K.A. Woodbury, Y. Chen and Q. Ke, Determination of interfacial heat fluxes during casting of aluminum in resin-bonded sand molds. *AFS Trans.* (1998) **106**, 98-116, 705-711.
8. T. Swada and N. Nishiwaki, Response of a thermocouple to transient temperature changes in a metal to which it is attached. *J. Mech. Sci.* (1991) **33**(7), 551-561.
9. J.V. Beck, B. Blackwell and C.R. St. Clair, Jr., *Inverse Heat Conduction*, John Wiley and Sons, New York, 1985.
10. www.fluent.com; FIDAP Manual.
11. E.M. Sparrow, Error estimates in temperature measurements, in *Measurements in Heat Transfer*, (eds. E.R.G. Eckert and R.J. Goldstein), Hemisphere, Berlin, 1976, pp. 1-23.